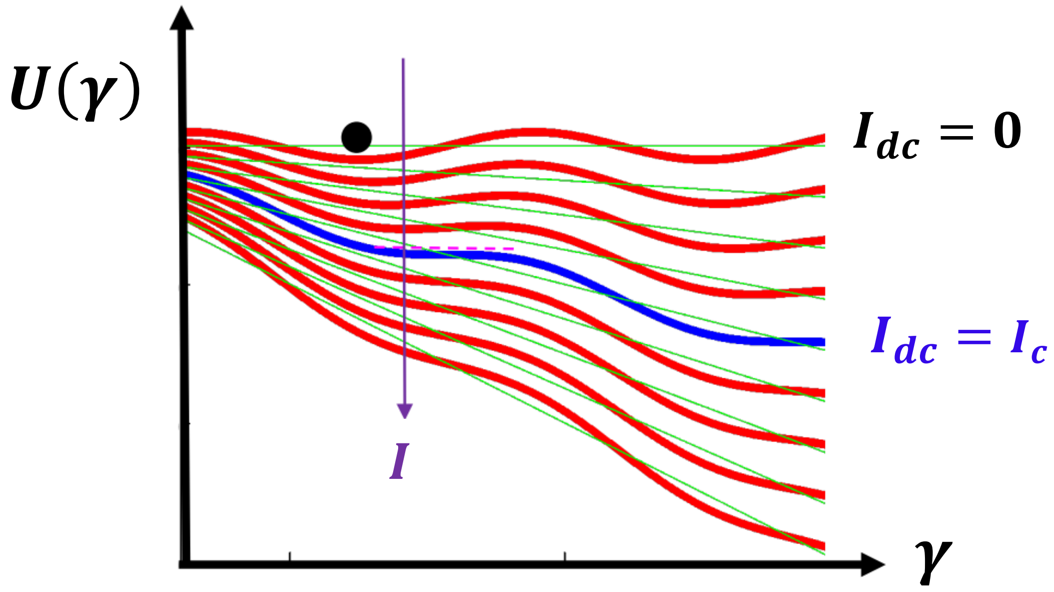
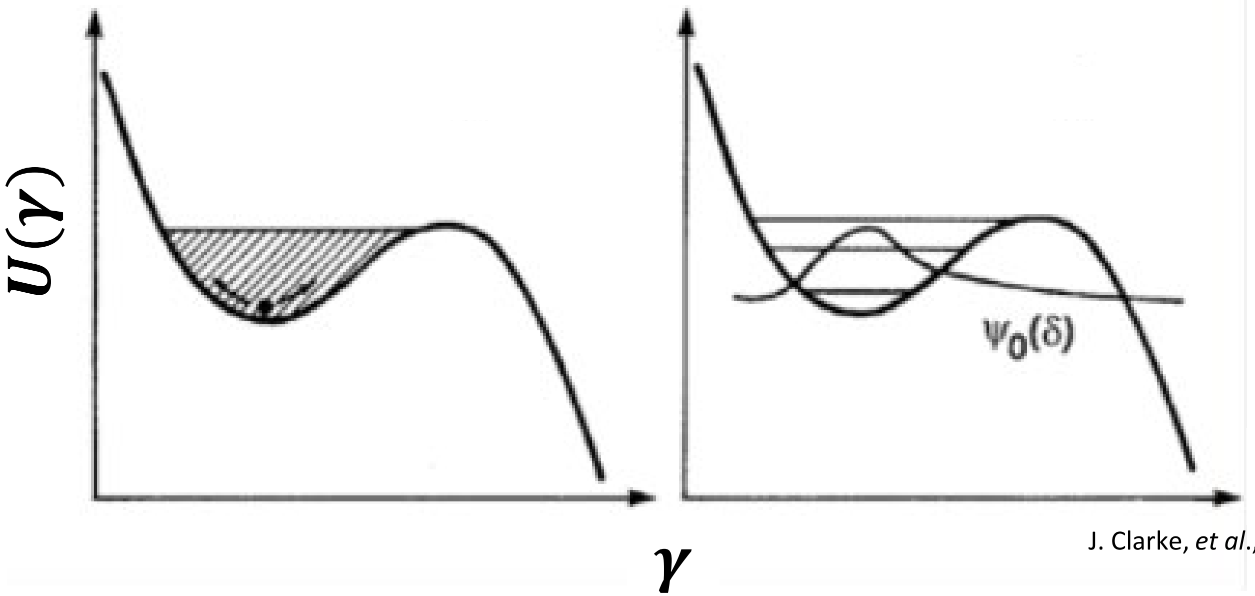
When is a Current-Biased Josephson Junction in the Quantum Limit?

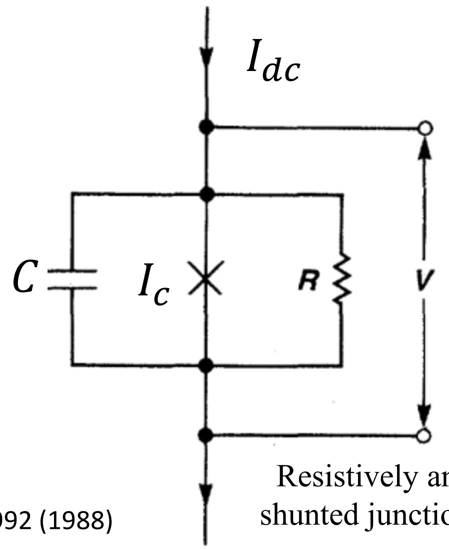
A current-biased Josephson junction has a potential given by $U(\gamma) = -\frac{\Phi_0}{2\pi}\,(I_{dc}\,\gamma + I_c \cos\gamma)$,
where γ is the gauge-invariant phase difference and Φ_0 is the flux quantum.
This creates the tilted washboard potential as a function of gauge-invariant phase.



When is the Problem Quantum as opposed to Classical?



J. Clarke, *et al.*, Science 239, 992 (1988)



Resistively and capacitively-shunted junction (RCSJ) model

DC Josephson equation
$\boldsymbol{I = I_c \sin\gamma}$

The gauge-invariant phase point oscillates in an (approximately) harmonic oscillator potential characterized by a curvature given by the dc-current-dependent Josephson plasma frequency $\omega_p(I_{dc})$:

$$E_n \approx \hbar\omega_p\left(n+\frac{1}{2}\right) - \text{small anharmonic corrections.} \qquad n = 0,1,2,3,\ldots$$

with

$$\omega_p = \sqrt{\frac{2eI_c}{\hbar C}}\left(1-\left(\frac{I_b}{I_c}\right)^2\right)^{1/4}$$

To see quantum transitions between the states the thermal energy must be small compared to the energy level spacing

$$\boldsymbol{k_B T \ll \hbar\omega_p}$$

To minimize the effects of loss we also require: $\boldsymbol{Q = \omega_p RC \gg 1}$, in the language of the RCSJ model, shown above

This approach leads to a phase qubit